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M2 Quantum, Light, Matter, Nanosciences

Fundamentals of Nanophotonics - Tutorial 4

Plasmonics

Surface plasmon excitation on a metallic grating

A surface plasmon polariton (SPP) propagating along a flat metal–vacuum interface is characterized by an in-plane wavevector

$$\mathbf{k}_{\parallel} = (k_x, k_y).$$

For a given angular frequency ω, the dispersion relation of the SPP reads

$$k_{\rm sp}(\omega) = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m(\omega)}{\varepsilon_m(\omega)+1}},$$

where $\varepsilon_m(\omega)$ is the dielectric function of the metal.

Since the interface is translationally invariant in the plane, the dispersion depends only on the magnitude of the in-plane wavevector. Thus,

$$k_x^2 + k_y^2 = k_{\rm sp}^2(\omega),$$

which is the equation of a circle of radius $k_{\rm sp}(\omega)$ in the (k_x, k_y) plane.

Grating-assisted excitation

An incident plane wave is characterized by its in-plane wavevector

$$\mathbf{k}_{\parallel,\rm inc} = (k_{x,\rm inc}, k_{y,\rm inc}).$$

On a flat surface, momentum conservation forbids direct excitation of the SPP. A periodic grating of period d along the x-axis introduces a reciprocal lattice vector

$$G = \frac{2\pi}{d}.$$

Momentum conservation parallel to the surface becomes

$$\mathbf{k}_{\parallel,\rm inc} + mG\,\hat{x} = \mathbf{k}_{\rm sp}, \qquad m \in \mathbb{Z}.$$

The condition to excite a surface plasmon is therefore

$$(k_{x,\rm inc} + mG)^2 + k_{y,\rm inc}^2 = k_{\rm sp}^2(\omega).$$

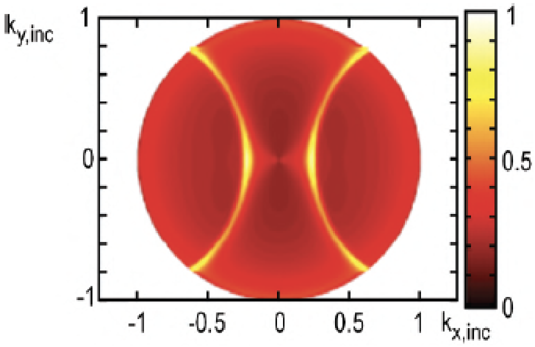

FIGURE 1 – p-polarized absorptivity of a gold grating with period d along the x-axis in the plane $(k_{x,\text{inc}}, k_{y,\text{inc}})$.

Solutions in reciprocal space

For each diffraction order m, this equation represents a circle of radius $k_{\text{sp}}(\omega)$ in the $(k_{x,\text{inc}}, k_{y,\text{inc}})$ plane, centered at $(-mG, 0)$. The full set of solutions consists of a series of circles periodically shifted along the k_x direction.

Absorptivity map

Surface plasmons are lossy electromagnetic modes in metals. Therefore, light absorption is enhanced whenever the incident wavevector satisfies the plasmon excitation condition. The absorptivity map in the $(k_{x,\text{inc}}, k_{y,\text{inc}})$ plane exhibits maxima along the shifted SPP circles corresponding to different diffraction orders m. The pattern is periodic along k_x with period $G = 2\pi/d$ and invariant along k_y, reflecting the one-dimensional periodicity of the grating.

Surface-plasmon resonance and electrostatic reflection factor

1. Reflection factor for p-polarization

For p-polarized light incident on a planar interface between vacuum ($\varepsilon_0 = 1$) and a non-magnetic material ($\mu = 1$) with dielectric function $\varepsilon_1(\omega)$, the Fresnel reflection factor is

$$r_p = \frac{\varepsilon_1 k_z - k_{z1}}{\varepsilon_1 k_z + k_{z1}},$$

where

$$k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}, \qquad k_{z1} = \sqrt{\varepsilon_1 \frac{\omega^2}{c^2} - k_x^2}.$$

2. Limit $k_x \gg \omega/c$

In the limit $k_x \gg \omega/c$, we have $k_z \approx ik_x$, and

$$k_{z1} \approx ik_x \quad (\text{assuming } \varepsilon_1 \text{ finite}).$$

The reflection factor becomes

$$r_p = \frac{\varepsilon_1(ik_x) - ik_x}{\varepsilon_1(ik_x) + ik_x} = \frac{\varepsilon_1 - 1}{\varepsilon_1 + 1}.$$

In this limit, r_p is indeed independent of k_x. The factor is exactly the reflection factor used in electrostatics to compute the amplitude of the *image charge* induced by a point charge near a planar interface.

Surface resonance using a Drude model

The non-lossy Drude model reads

$$\varepsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega^2},$$

where ω_p is the plasma frequency. A surface resonance occurs when

$$r_p \to \infty \quad \Rightarrow \quad \varepsilon_1(\omega_s) + 1 = 0.$$

Solving for ω_s :

$$1 - \frac{\omega_p^2}{\omega_s^2} + 1 = 0 \quad \Rightarrow \quad \omega_s = \frac{\omega_p}{\sqrt{2}}.$$

Thus, the surface plasmon resonance occurs at

$$\boxed{\omega_s = \frac{\omega_p}{\sqrt{2}}}.$$

For a lossy Drude model :

$$\varepsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)},$$

with damping rate γ. The surface resonance condition is

$$\varepsilon_1(\omega_s) + 1 = 0 \quad \Rightarrow \quad 1 - \frac{\omega_p^2}{\omega_s(\omega_s + i\gamma)} + 1 = 0.$$

This leads to a complex resonance frequency :

$$\omega_s = \frac{\omega_p}{\sqrt{2}} - i\frac{\gamma}{2}.$$

The imaginary part $-i\gamma/2$ represents the **decay rate** of the surface plasmon due to losses in the metal. It corresponds to the finite lifetime of the oscillation.

Quality factor of the oscillation

The quality factor Q is defined as

$$Q = \frac{\Re(\omega_s)}{2|\Im(\omega_s)|} = \frac{\omega_p/\sqrt{2}}{\gamma}.$$

This measures the number of oscillations before the surface plasmon decays significantly.

$$Q = \frac{\omega_p}{\sqrt{2}\,\gamma}$$