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M2 Quantum, Light, Matter, Nanosciences

Fundamentals of Nanophotonics - Tutorial 2

Integral formulation - Extinction theorem.

Computing Fresnel factors using the integral formalism

The purpose of this exercise is to derive the Fresnel reflection factor by solving the integral equation for an interface-separating vacuum from a homogeneous medium. It will be seen that the field radiated by the induced polarization cancels the incident field inside the medium (extinction theorem) on the one hand and generates both the transmitted and the reflected field on the other. The half-space $z < 0$ is filled with a non-magnetic medium with complex permittivity ϵ_r describing either a dielectric or a metal. It is illuminated by a plane wave with the electric field polarized along Oy (see Fig. 12.9).

We want to derive the reflection and transmission amplitudes by performing a radiation calculation. We first need to find the field radiated by the current density induced in the medium. The current density is thus

$$\mathbf{j} = i\omega\epsilon_0(\epsilon_r - 1)\mathbf{E}$$

The permittivity $\epsilon_r = n^2$ is the square of the refractive index n for a non-magnetic medium.

(1) Briefly recall the expression of the Fresnel transmission and reflection coefficients (in amplitude) at normal incidence. For a plane wave excitation at normal incidence, write the expressions of the reflected and transmitted wave at a given position z using these coefficients.

(2) Derive the form of the *transmitted* field in the medium at a depth z, noting that it is the sum of the fields radiated by three types of sources :

1. the current density in the source which is generating the incident field ;
2. the induced current density with $z' > z$, i.e. from above the studied plane ;
3. the induced current density with $z' < z$.

Quickly justify that the divergence of the electric field is here zero. Then, using the special result,

$$\int \frac{\exp[i\frac{\omega}{c}|\mathbf{r} - \mathbf{r}'|]}{|\mathbf{r} - \mathbf{r}'|} \mathrm{d}x'\mathrm{d}y' = \frac{2i\pi c}{\omega} \exp[i\frac{\omega}{c}|z - z'|],$$

derive contributions 2. and 3. in the form of an integral depending on the unknown electric field in the medium $\mathbf{E}(z')$.

(3) The previous equation is an integral equation for the field $\mathbf{E}(z)$. We seek a solution in the form of a plane wave :

$$\mathbf{E}(z) = A\exp(-iKz')\mathbf{u_y}$$

with $\mathrm{Im}(K) > 0$. Denoting $K = n\omega/c$, determine the form of A. Compare with the transmission factor given by the Fresnel factor.