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M2 Quantum, Light, Matter, Nanosciences

Fundamentals of Nanophotonics - Tutorial 2

Integral formulation

We consider an interface at $z = 0$ separating vacuum ($z > 0$) from a homogeneous, non-magnetic medium ($z < 0$) of relative permittivity $\varepsilon_r = N^2$ (N complex in general). The incident plane wave propagates along $-z$ and is polarized along $\hat{\mathbf{y}}$.

The induced current density inside the medium is

$$\mathbf{j}(\mathbf{r}) = -i\omega\varepsilon_0(\varepsilon_r - 1)\mathbf{E}(\mathbf{r}). \tag{1}$$

Throughout the exercise, fields have harmonic time dependence $e^{-i\omega t}$.

1. Fresnel reflection and transmission amplitudes

For a planar interface between vacuum ($n_1 = 1$) and a non-magnetic medium of refractive index N, the Fresnel coefficients at normal incidence are

$$r = \frac{1-N}{1+N}, \tag{2}$$

$$t = \frac{2}{1+N}. \tag{3}$$

The reflected and transmitted electric fields are therefore

$$\mathbf{E}_r(z > 0) = rE_0\, e^{+ik_0 z}\, \hat{\mathbf{y}}, \tag{4}$$

$$\mathbf{E}_t(z < 0) = tE_0\, e^{-ik_0 N z}\, \hat{\mathbf{y}}, \tag{5}$$

with $k_0 = \omega/c$.

2. Integral formulation of the transmitted field

We now derive the transmitted field using the radiation integral formalism. At a point $z < 0$ inside the medium, the total electric field is the sum of three contributions :

1. the field radiated by the external source that generates the incident wave ;
2. the field radiated by induced currents located at $z' > z$;
3. the field radiated by induced currents located at $z' < z$.

(a) Contribution of the external source The external source produces the incident plane wave

$$\mathbf{E}^{(a)}(z) = E_0 e^{-ik_0 z}\, \hat{\mathbf{y}}. \tag{6}$$

This field exists everywhere in space and would persist inside the medium if no induced currents were present.

(b) Contribution of induced currents with $z' > z$ The electric field radiated at point z by a current sheet located at z' is

$$dE_y(z) = \frac{i\omega\mu_0}{2k_0} j_y(z') e^{ik_0|z-z'|}\, dz'. \tag{7}$$

For $z' > z$, $|z - z'| = z' - z$, hence

$$E^{(b)}(z) = \frac{i\omega\mu_0}{2k_0} \int_z^0 j_y(z')\, e^{ik_0(z'-z)}\, dz'. \tag{8}$$

Substituting the induced current density,

$$E^{(b)}(z) = \frac{\omega^2}{2c^2k_0}(\varepsilon_r - 1) \int_z^0 E(z')\, e^{ik_0(z'-z)}\, dz'. \tag{9}$$

(c) Contribution of induced currents with $z' < z$ For $z' < z$, $|z - z'| = z - z'$, yielding

$$E^{(c)}(z) = \frac{i\omega\mu_0}{2k_0} \int_{-\infty}^{z} j_y(z')\, e^{ik_0(z-z')}\, dz'. \tag{10}$$

Substituting again the induced current,

$$E^{(c)}(z) = \frac{\omega^2}{2c^2k_0}(\varepsilon_r - 1) \int_{-\infty}^{z} E(z')\, e^{ik_0(z-z')}\, dz'. \tag{11}$$

Total field and extinction theorem

The total field inside the medium is therefore

$$E(z) = E^{(a)}(z) + E^{(b)}(z) + E^{(c)}(z). \tag{12}$$

Solving this integral equation with the ansatz

$$E(z) = E_t e^{-ik_0Nz}, \tag{13}$$

one finds that :

- the incident field $E^{(a)}$ is exactly canceled inside the medium by part of the radiated field (extinction theorem) ;
- the remaining field corresponds to the transmitted wave with wavevector k_0N ;
- the radiation toward $z > 0$ reconstructs the reflected wave with Fresnel amplitude r.

Thus, the integral formalism reproduces exactly the Fresnel reflection and transmission coefficients and provides a microscopic interpretation of reflection, transmission, and extinction.