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M2 Quantum, Light, Matter, Nanosciences

Fundamentals of Nanophotonics - Tutorial 1

Near field optics of a diffraction grating

A one-dimensional transmission grating with period Λ imposes a spatial modulation on an incident optical field. For an incident monochromatic plane wave of free-space wavelength λ at normal incidence, the transmitted field at $z = 0^+$ can be written as a Fourier series :

$$E(x, z = 0^+) = \sum_{m=-\infty}^{+\infty} T_m \, e^{i(k_x + mG)x}, \tag{1}$$

where

$$G = \frac{2\pi}{\Lambda} \quad \text{(grating vector)}, \tag{2}$$

$$T_m = \frac{1}{\Lambda} \int_0^{\Lambda} t(x) e^{-imGx} \, \mathrm{d}x \quad \text{(Fourier coefficients of the transmission function)}. \tag{3}$$

Fourier expansion of a binary grating

Consider a binary amplitude grating with period Λ and a 50% duty cycle defined by

$$t(x) = \begin{cases} 1, & 0 < x < \dfrac{\Lambda}{2}, \\ 0, & \dfrac{\Lambda}{2} < x < \Lambda. \end{cases} \tag{4}$$

1. Compute the Fourier coefficients

$$T_m = \frac{1}{2} \operatorname{sinc}\left(\frac{m\pi}{2}\right) = \frac{1}{2} \cdot \frac{\sin\left(\frac{m\pi}{2}\right)}{\frac{m\pi}{2}} \quad (m \neq 0), \tag{5}$$

   and $T_0 = 1/2$.
2. Write the transmitted field immediately after the grating as a sum of plane waves using the coefficients T_m.

Propagating vs Evanescent orders

3. Write a propagation condition for the order m when the grating is illuminated under normal incidence. Define a cut-off order.
4. Provide a short physical explanation of why decreasing Λ causes more orders to become evanescent.
5. For a grating embedded between two dielectrics with refractive indices n_1 (incidence side) and n_2 (exit side), write the modified propagation condition for the exit side.

We now consider illumination by a HeNe laser with wavelength $\lambda = 633\,\mathrm{nm}$. Study three gratings :

1. $\Lambda = 10\,\mu\mathrm{m}$,
2. $\Lambda = 1\,\mu\mathrm{m}$,
3. $\Lambda = 300\,\mathrm{nm}$.

For each grating :

1. Compute G and determine all integers m for which the diffracted order is propagating in free space.
2. Sketch the angular distribution of propagating orders. Use the grating equation for transmission angles.
3. Identify the regime : "many propagating orders (coarse grating)", "few propagating orders (intermediate)", or "subwavelength grating (only m=0 propagating)".

Near-field region

1. For $\Lambda = 300\,\mathrm{nm}$, compute the decay constant of the first evanescent order $m = \pm 1$:

$$\alpha_m = \sqrt{(mG)^2 - k_0^2}, \tag{6}$$

   so that the field decays as $e^{-\alpha_m z}$. Express your result in nm^{-1} and estimate the $1/e$ decay length.
2. Discuss qualitatively strategies to collect evanescent waves with a detector in the far-field.

Optional challenge — Arbitrary transmission function

Let the grating transmission be a Gaussian aperture array :

$$t(x) = \exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right] \sum_{n=-\infty}^{\infty} \delta(x - n\Lambda). \tag{7}$$

1. Derive the Fourier coefficients analytically (use the Poisson summation formula) and show how the Gaussian envelope controls the amplitude of diffracted orders.
2. Discuss how changing σ affects the envelope and thus the number of strong diffracted orders.

Key conceptual questions

1. What determines whether a diffracted order is propagating or evanescent ?
2. How does the grating period affect the angular distribution of the transmitted field ?
3. Why are evanescent waves important in near-field optics and nanophotonics ?