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M2 Quantum, Light, Matter, Nanosciences

Fundamentals of Nanophotonics - Tutorial 1
SOLUTION
Near field optics of a diffraction grating

Fourier expansion of a binary grating

(a) Fourier coefficients

The grating transmission is

$$t(x) = 1 \text{ for } 0 < x < \Lambda/2, \qquad t(x) = 0 \text{ for } \Lambda/2 < x < \Lambda.$$

Thus

$$T_m = \frac{1}{\Lambda}\int_0^{\Lambda/2} e^{-imGx}\,dx \tag{1}$$

$$= \frac{1}{\Lambda}\left[\frac{e^{-imGx}}{-imG}\right]_0^{\Lambda/2} \tag{2}$$

$$= \frac{1}{\Lambda}\frac{1-e^{-imG\Lambda/2}}{imG}. \tag{3}$$

Since $G\Lambda = 2\pi$,

$$T_m = \frac{1}{\Lambda}\frac{1-e^{-im\pi}}{imG} = \frac{1}{\Lambda}\frac{1-(-1)^m}{imG}.$$

For odd m, $1-(-1)^m = 2$; for even m, it vanishes. Thus

$$T_m = \frac{1}{2}\text{sinc}\left(\frac{m\pi}{2}\right), \qquad T_0 = 1/2.$$

(b) Transmitted field

$$E(x, z = 0^+) = \sum_{m=-\infty}^{+\infty} T_m e^{imGx}.$$

Only odd orders contribute for this grating.

Propagating vs evanescent orders

A propagating order requires

$$k_{z,m} = \sqrt{k_0^2 - (mG)^2} \in \mathbb{R}.$$

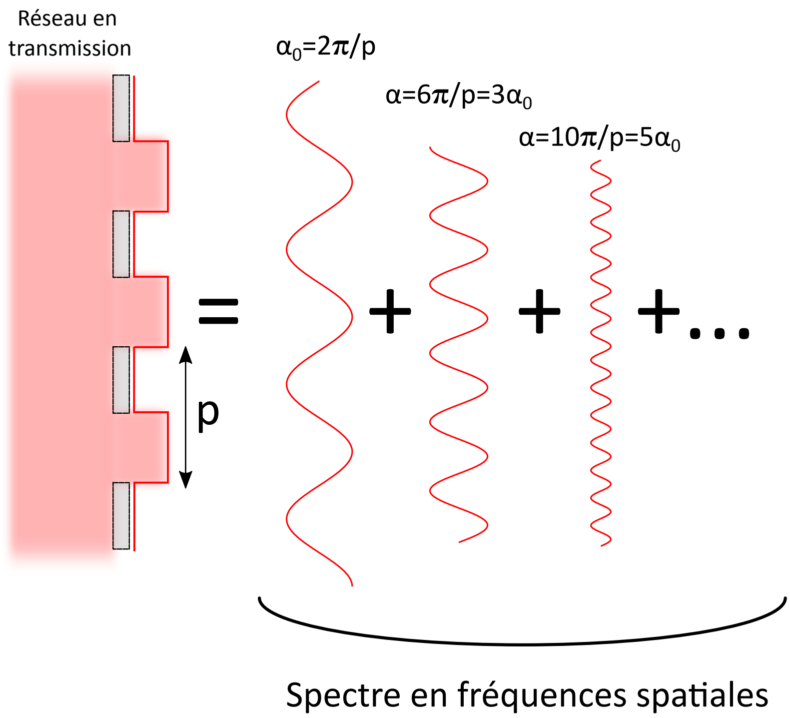

FIGURE 1 – Spatial frequency decomposition of the plane wave spectrum after transmission by a grating.

Thus propagation requires $|m|G \leq k_0$, leading to cutoff

$$m_{\max} = \left\lfloor \frac{k_0}{G} \right\rfloor = \left\lfloor \frac{\Lambda}{\lambda} \right\rfloor .$$

Orders with $|m| > m_{\max}$ yield imaginary $k_{z,m}$ and are evanescent.

Physically, decreasing Λ increases $G = 2\pi/\Lambda$, spacing the diffraction orders further apart in momentum space so that only $m = 0$ may remain inside the propagating light cone.

If the transmitted medium has refractive index n_2, propagation requires

$$|m|G \leq n_2 k_0.$$

Three grating regimes

Given $\lambda = 633$ nm.

Case 1 : $\Lambda = 10\,\mu$m

$$G = \frac{2\pi}{10\,\mu\text{m}} = 0.628\,\mu\text{m}^{-1}, \qquad k_0 = \frac{2\pi}{0.633\,\mu\text{m}} = 9.93\,\mu\text{m}^{-1}.$$

Propagation requires $|m| \leq 15$. Thus many orders propagate.

Angles follow

$$\sin\theta_m = m\frac{\lambda}{\Lambda} = m \times 0.0633.$$

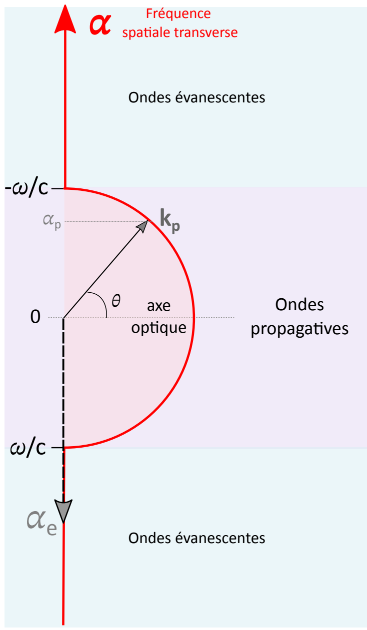

FIGURE 2 – Limit between evanescent waves and propagating waves.

Case 2 : $\Lambda = 1\,\mu$m

$$G = 6.28\,\mu\text{m}^{-1}.$$

Propagation : $|m| \le 1$ (since $1 \cdot G < k_0$ but $2 \cdot G > k_0$). Only $m = -1, 0, 1$ propagate.

Angles :

$$\sin\theta_{\pm 1} = \pm 0.633.$$

Case 3 : $\Lambda = 300$ nm

$$\Lambda = 0.3\,\mu\text{m}, \quad G = 20.94\,\mu\text{m}^{-1} > k_0.$$

Thus even $m = \pm 1$ are evanescent; only $m = 0$ propagates.

Evanescent decay

For $\Lambda = 300$ nm,

$$G = 20.94\,\mu\text{m}^{-1}, \qquad k_0 = 9.93\,\mu\text{m}^{-1}.$$

For $m = 1$:

$$\alpha_1 = \sqrt{G^2 - k_0^2} = \sqrt{(20.94)^2 - (9.93)^2} = 18.6\,\mu\text{m}^{-1}.$$

Decay length :

$$L_{1/e} = \frac{1}{\alpha_1} \approx 54\,\text{nm}.$$

Thus energy is concentrated in the near field.

Near-field effect : subwavelength gratings strongly reshape evanescent waves, which influence fields within tens of nm, though no far-field diffraction is visible.

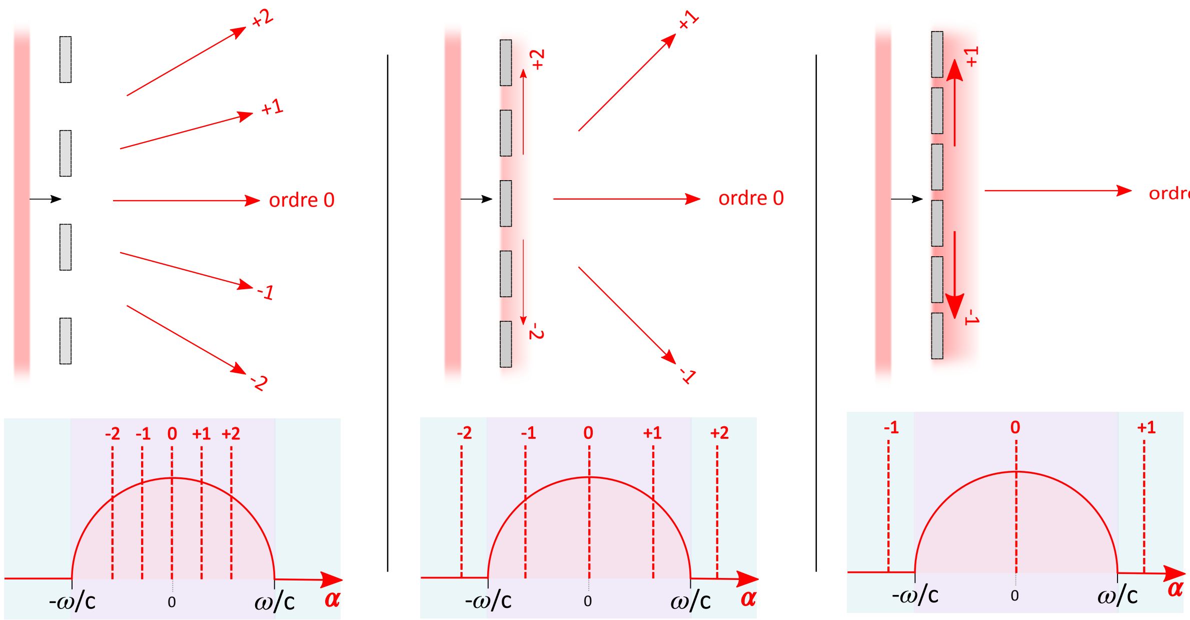

FIGURE 3 – Three diffraction regimes for a periodic grating.