Benjamin Vest
benjamin.vest@institutoptique.fr
Correction au 13 octobre 2021

TD 4 Physique Statistique Hors équilibre
Introduction to transport phenomena

1 Diffusion current in a p-n junction

In this exercise, we will focus on the situation of a p-n junction at equilibrium. Equilibrium does not mean that the system is "frozen" : the system still freely evolves and fluctuates around states displaying similar variables.

In particular, fluxes can exist between different sub systems, but the net total flux flowing through a subsystem is in average zero. We can use this definition of equilibrium to compute the amplitude of transport phenomena within a system at equilibrium.

Number of electrons per unit volume. Electrons in a semiconductor occupy states in a band structure, following the Fermi-Dirac distribution. In our situation, we are mostly interested by conduction electrons, which are responsible for current flowing through the semiconductor. We can immediately write :

$$n = \int_0^{\infty} g(E) \frac{1}{e^{\frac{E-\mu}{kT}} + 1} dE$$

where $g(E)$ is the density of states in the conduction band, defined by the nature of the semiconductor itself.

Classical limit. In the classical limit, we consider that the occupation numbers of the considered states are much smaller than 1. As a consequence, there is no quantum effect observable in the statistics and distribution of paritcle (no electrons being repelled to higher energy states by Pauli's principle for example). We also can simplify the previous expression :

$$n = \int_0^{+\infty} g(E) e^{\frac{\mu-E}{kT}} dE$$

p-n junction at equilibrium . When building a junction, we put in contact two doped regions. Taken independently, these two junctions, at rest, exhibit radically different carriers densities. Quite logically, the p region displays a majority of positively charged holes, and the n region a majority of negatively charged conduction electrons.

In a junction, the density of electrons is therefore far from constant. Far from the interface indeed, we must retrieve the properties of the "bulk" doped pieces of semiconductor, with no influence of the other region whatsoever.

For the same reason, the chemical potential of electrons in the junction at equilibrium is not constant. This means that a diffusion current j_{diff} must exist along the junction, describing a flow of electrons from the higher densities of electrons to the lower densities of electrons.
However, the definition of equilibrium implies that the total flux of electrons, hence the total current j_{tot} is zero along the junction. This means that another mechanism exists, and compensate the diffusion current.

This contribution is the *ohmic drift current* j_{drift} generated by the presence of an electric field in the depletion region, itself due to the presence of dopants. We can write :

$$j_{diff} = \sigma \mathbf{E}$$

where σ is the electrical conductivity, and $\mathbf{E}$ the electric field in the depletion region.

Derivation of the diffusion current. We use kinetic theory arguments to derive the diffusion current. We consider a cross-section Σ of the junction. The diffusion current is related to the net flux of particles flowing through this cross section per unit of time. The number of particles crossing the line from each side depends on the number of electrons close enough from the border to actually cross it, and whose velocity is correctly oriented. Therefore we write :

$$\begin{aligned} j_d &= -e\frac{1}{6}\left(n(x-l)v - n(x+l)v\right) \\ &= \frac{evl}{3}\frac{\partial n}{\partial x} \\ &= \frac{ekT}{m_e}\tau\frac{\partial n}{\partial x} \end{aligned}$$

Derive the induced field. Simply using the equilibrium condition$j_{tot} = 0 = \sigma E + j_{diff}$:

$$E = \frac{-1}{\sigma} j_{diff} = \frac{-m_e}{ne^2\tau}\frac{kTe\tau}{m_e}\frac{\partial n}{\partial x} = \frac{-kT}{ne}\frac{\partial n}{\partial x}$$

where we used the expression of the conductivity $\sigma = \frac{ne^2\tau}{m}$

2 Viscosity

Viscosity is a measure of internal friction between adjacent layers of a fluid. This friction can be seen as a transport of momentum between layers by particles in the fluid. In this exercise, we consider a laminar flow $V_x(z)\mathbf{e_x}$.
(the liquid flows in the x-direction, but layers at different$z$ can have a different velocity). We use elements of the kinetic theory to solve the problem.

Viscosity μ is given by $F_{xz} = -\mu\frac{\partial V_x}{\partial z}$. Let us consider of small volume, defined by a cross-section Σ separating two layers and a lateral extension zd. The net momentum transfer in this layer is given by contributions from upper and lower layers closer than the free mean path, and from particles with appropriate velocities. We consider that particles have a random thermal velocity v, same in all directions, that adds up to the collective laminar flow velocity V_x.

$$\begin{aligned} \frac{dp_{syst,x}}{dt} &= \Sigma\frac{1}{6}\left(nv_z[mV_x(z-l)] - nv_z[mV_x(z+l)]\right) \\ &= -\Sigma\frac{n}{6}vml2\frac{\partial V_x}{\partial z} \end{aligned}$$

The shear force is a force per unit surface. We divide the previous momentum transfer by Σ and use the other expressions of the mean free path $l = v\tau$ and root mean square velocity $v^2 = \frac{3kT}{m}$ to get :

$$F_{xz} = -nkT\tau\frac{\partial V_x}{\partial z}$$

and we identify directly the viscosity : $\mu = nkT\tau$.