Master QLMN, CentraleSupelec

Non Equilibrium Statistical Physics

Transport Phenomena

1 Boltzmann Equation. Viscosity

We consider a laminar flow with a velocity field $\mathbf{V}(z) = V_x(z)\mathbf{e_x}$. The fluid is described by the Maxwell-Boltzmann velocity distribution :

$$f^0(\mathbf{r},\mathbf{v}) = n\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left[-\frac{m(\mathbf{v}-\mathbf{V}(z))^2}{2k_B T}\right] \tag{1}$$

We study the transfer of momentum p_x through the plane $z = z_0$ using a kinetic description. We want to show that the flux of momentum per unit area can be written as :

$$\Phi_{p_x} = P_{xz} = -\mu\frac{\partial V_x}{\partial z} \tag{2}$$

1. Write the general expression for the flux density of momentum.

2. Derive the expression of the distribution of velocity using the relaxation time approximation.

3. Derive the viscosity μ.

2 Boltzmann Equation. Fick's Law of Diffusion

We consider an undoped semiconductor material. The number of electrons in the conduction band is small. We can consequently use the Maxwell-Boltzmann distribution. We want to derive the diffusion coefficient D for an electron so that the flux of electron is :

$$\mathbf{j}_n = -D\nabla n(\mathbf{r}).$$

1. Write the general expression of the diffusion flux $\mathbf{j}_n$ as a function of the number of electrons per unit volume in the phase space $(\mathbf{r},\mathbf{v})$: $f(\mathbf{r},\mathbf{v})$.

2. Derive the concentration of electrons $n(\mathbf{r})$ as a function of $f(\mathbf{r},\mathbf{v})$.

3. There is a stationary gradient of electron concentration in the system. Using Boltzmann Equation in the relaxation time approximation, calculate the distribution function of velocity. We make the hypothesis that the system is close to equilibrium. The relaxation time of the system is τ.

4. Derive Fick's law and the diffusion coefficient D.