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Master QLMN - Université Paris-Saclay

TD 4 Non-Equilibrium Statistical Physics
Introduction to transport phenomena

1 Diffusion Current in a p-n Junction

1. Write the number of electrons per unit volume as a function of the density of states $g(E)$, the chemical potential μ and the Fermi-Dirac distribution.

2. Simplify this form using the classical limit.

In this limit, compute the value of the quadratic mean velocity $v^2 = \langle(\vec{v})^2\rangle$ assuming the electron gas to be a perfect gas. Recall that $g(E)dE \leftrightarrow d\vec{r}d\vec{p}/h^3$ and that

$$\frac{\int x^2 \exp(-ax^2 dx)}{\int \exp(-ax^2)dx} = 1/2a$$

Plot the distribution of velocities along one axis, say v_x, and the distribution of the modulus of $\vec{v}$.

3. We consider a p-n junction in equilibrium with no applied voltage (open circuit).Is the density of electrons constant along the junction ? Is the chemical potential constant along the junction ? Is the current density non zero in the junction ? What is the mechanism that compensates for the diffusion current ?

4. Derive the diffusion current in the depletion region as a function of the electron density.

l is the mean free path, v^2 the quadratic mean square velocity, τ the average time between consecutive collisions, m_e the electron mass and $-e$ its charge. We assume that $v^2 = 3k_BT/m_e$.

5. Derive the induced electric field at equilibrium. Deduce an expression of the electrical conductivity σ.

2 Viscosity

We consider a laminar flow with a velocity field $V_x(z)\mathbf{e_x}$. We will derive the viscosity of the fluid that will be considered as incompressible, isothermal and composed of identical molecules of mass m.

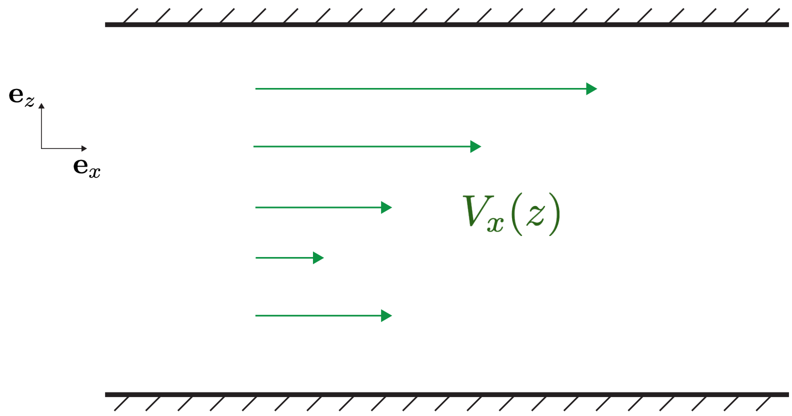

FIGURE 1 – A laminar flow of velocity $\vec{V} = V_x(z)\mathbf{e_x}$ is oriented along the x-axis with an amplitude varying along the z-axis.

1. Derive the transfer of $\mathbf{e_x}$-oriented momentum across the plane $z = z_0$. We introduce the quadratic mean velocity $v^2 = 3k_BT/m$ and the mean free path $l = v\tau$ where τ is the collision time.

2. Starting from a momentum balance on the half-space $z > 0$, show that the shear stress (force per unit surface) can be cast in the form :

$$F_{xz} = -\mu \frac{\partial V_x}{\partial z} \tag{1}$$

Derive the viscosity μ as a function of n, k_B, T, τ and m.