Master Nanosciences, parcours Nanophysique

Non Equilibrium Statistical Physics

1 Fluctuation of Polarizability for a Dielectric Particle. Langevin Model

We consider a dielectric particle much smaller than the excitation wavelength. In presence of a static electric field $\mathcal{E}_0$, the particle has a dipole moment $\mathbf{p}_0$ given by its static susceptibility χ_0. When the electric field is switched off, the amplitude of the dipole moment decays exponentially with a time constant $\tau = 1/\gamma$.

1. Write the differential equation that describes the exponential decay of the dipole moment (external field switched off).

2. Write the same equation including a noise term $\mathbf{f}$ that causes fluctuations. We use the assumption that $< f_i(t) f_j(t+\tau) >= |f|^2 \delta_{ij} \delta(\tau)$. Explain this approximation.

3. Write the expression of the power spectral density for the fluctuations of dipole moment. Use this expression to derive the correlation function $< p_i(t) p_i(0) >$ as a function of $|f|^2$. At equilibrium, the correlation is $< p_i(0) p_i(0) >= k_B T \chi_0$. What is the value of $|f|^2$? Derive the expression of the power spectral density of the dipole moment fluctuations.

2 Fluctuations of Elongation for a Spring. Langevin Model

The goal of the exercise is to model the time correlation of the fluctuations of a spring elongation. Such a model can be used to describe the thermal fluctuations of the position of the tip of an atomic force microscope. We use the hypothesis that the force applying to the string is given by a viscous term $-\gamma m \dot{x}$.

1. Write the equation of motion of the string in harmonic regime, including a noise term R.

2. Derive the power spectral density for the fluctuation of position I_x as a function of the power spectral density for the fluctuations of the force I_R.

3. Assuming I_R is constant with respect to frequency, derive an integral expression of the temporal correlation function $< x(t) x(t+\tau) >$. Calculate the integral.