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Master QLMN - Université Paris-Saclay

TD 3 Non-Equilibrium Statistical Physics

Langevin model

1 Fluctuations of polarization of a dielectric particle. Langevin Model

We consider a dielectric particle much smaller than the excitation wavelength. In presence of a static electric field $\mathcal{E}_0$, the particle has a dipole moment $\mathbf{p}_0$ given by its static susceptibility χ_0. When the electric field is switched off, the amplitude of the dipole moment decays exponentially with a time constant $\tau = 1/\gamma$.

1. Write the differential equation that describes the exponential decay of the dipole moment (external field switched off).

2. Write the same equation including a vectorial noise term $\mathbf{f}$ that causes fluctuations. We use the assumption that $< f_i(t) f_j(t') >= \frac{I_f}{3}\delta_{ij}\delta(t-t')$. Explain this approximation.

3. Write the expression of the power spectral density for the fluctuations of the dipole moment. Use this expression to derive the correlation function $< p_i(t)p_i(0) >$ as a function of I_f. At equilibrium, the correlation is $< p_i(0)p_i(0) >= \frac{1}{3}k_B T \chi_0$. What is the value of I_f ? Derive the expression of the power spectral density of the dipole moment fluctuations.

2 Fluctuations of elongation of a mechanical harmonic oscillator. Langevin Model

The goal of the exercise is to model the time correlation of the fluctuations of the position of a mass m attached to a spring. Such a model can be used to describe the thermal fluctuations of the position of the tip of an atomic force microscope. We use the hypothesis that the force applying to the mass is given by a viscous term $-\gamma m \dot{x}$. The spring constant is denoted $K = m\omega_0^2$.

1. Write the equation of motion in time domain and in the harmonic regime, including a noise term R.

2. Derive the power spectral density for the fluctuation of position I_x as a function of the power spectral density for the fluctuations of the force I_R.

3. Assuming I_R is constant with respect to frequency, derive an integral expression of the temporal correlation function $< x(t)x(t+\tau) >$. Calculate the integral using the residue theorem.