

Non Equilibrium Statistical Physics - TD 2

1 Fluctuation-Dissipation theorem : polarizability of a dielectric Particle

We consider a dielectric particle much smaller than the excitation wavelength. In presence of a static electric field $\vec{\mathcal{E}}_0$, the particle has a dipole moment \vec{p}_0 given by its static susceptibility χ_0 ,

$$\langle \vec{p}_0 \rangle = \chi_0 \vec{\mathcal{E}}_0$$

When the electric field is switched off, the amplitude of the dipole moment decays exponentially with a time constant $\tau = 1/\gamma$.

Complex susceptibility

- 1) Derive the relaxation function $\Psi(t)$ and the linear response $\chi(t)$
- 2) Show that the complex susceptibility is $\chi(\omega) = \frac{\chi_0}{1-i\omega\tau}$

Fluctuations of the dipole moment

- 3) Using $\Psi(t)$, derive the temporal correlation function for the dipole moment
- 4) Plot the correlation function from $t = -\infty$ to $t = +\infty$
- 5) Applying the fluctuation-dissipation theorem, derive the power spectral density of the dipole moment fluctuations.

2 Kappler's experiment

In order to study experimentally the probability distribution in the canonical ensemble, Kappler measured the thermal fluctuations of the angular position of a mirror suspended by a torsion wire. The system is modeled by its moment of inertia J , the torsion coefficient C and the mirror rotation angle θ . The conjugate variables in phase space are the angle θ and the angular momentum $p = J\dot{\theta}$.

2.1 Equilibrium fluctuations

- 1) Define the state of the system in a classical framework.
- 2) The energy of the system is $\frac{J\dot{\theta}^2}{2} + \frac{C\theta^2}{2}$. Using energy conservation, derive the equation of motion of the oscillator. What is the frequency of the oscillator?
- 3) What is the classical approximation validity condition?
- 4) What is the canonical probability for the system to be in state (θ, p) within $d\theta dp$? Write the partition function using T , J and C .
- 5) Derive the mean energy from the expression of the partition function you just obtained from 4).
- 6) Calculate the quadratic mean value of the angle θ , $\langle \theta^2 \rangle$. Comment.

2.2 Temporal correlation of fluctuations from the Fluctuation-Dissipation theorem

- 1) Write the equation of motion in harmonic regime including fluid friction damping $-\gamma J\dot{\theta}$ and an external driving torque $C_0 \exp(-i\omega t)$.
- 2) Derive the susceptibility $\chi(\omega)$ and the power spectral density $I_\theta(\omega)$ for the fluctuations of θ .
- 3) Derive the temporal correlation function $C_{\theta\theta}(t)$ of the angular fluctuations in integral form.