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# TD 2 Non-Equilibrium Statistical Physics

# Fluctuation-Dissipation Theorem

## 1 Polarizability of a dielectric Particle

We consider a dielectric particle much smaller than the excitation wavelength. In presence of a static electric field $\vec{\mathcal{E}_0}$, the particle has a dipole moment $\vec{p_0}$ given by its static susceptibility $\chi_0$,

$$\langle \vec{p_0} \rangle = \chi_0 \vec{\mathcal{E}_0}$$

When the electric field is switched off, the amplitude of the dipole moment decays exponentially with a time constant $\tau = 1/\gamma$.
*Complex susceptibility*
1) Derive the relaxation function $\Psi(t)$ and the linear response $\chi(t)$
2) Show that the complex susceptibility is $\chi(\omega) = \frac{\chi_0}{1-i\omega\tau}$
*Fluctuations of of the dipole moment*
3) Using $\Psi(t)$, derive the temporal correlation function for the dipole moment
4) Plot the correlation function from $t = -\infty$ to $t = +\infty$
5) Applying the fluctuation-dissipation theorem, derive the power spectral density of the dipole moment fluctuations.

## 2 Kappler's experiment

In order to study experimentally the probability distribution in the canonical ensemble, Kappler measured the thermal fluctuations of the angular position of a mirror suspended by a torsion wire. The system is modeled by its moment of inertia $J$, the torsion coefficient $C$ and the mirror rotation angle $\theta$. The conjugate variables in phase space are the angle $\theta$ and the angular momentum $p = J\dot{\theta}$.

### 2.1 Equilibrium fluctuations

1) Define the state of the system in a classical framework.
2) The energy of the system is $\frac{J\dot{\theta}^2}{2} + \frac{C\theta^2}{2}$ Using energy conservation, derive the equation of motion of the oscillator. What is the frequency of the oscillator ?
3) What is the classical approximation validity condition ?
4) What is the canonical probability for the system to be in state $(\theta, p)$ within $d\theta dp$ ? Write the partition function using T, $J$ and $C$.
5) Derive the mean energy from the expression of the partition function you just obtained from 4).
6) Calculate the quadratic mean value of the angle $\theta$, $\langle \theta^2 \rangle$. Comment.

### 2.2 Temporal correlation of fluctuations from the Fluctuation-Dissipation theorem

1) Write the equation of motion in harmonic regime including fluid friction damping $-\gamma J\dot{\theta}$ and an external driving torque $C_0 \exp(-i\omega t)$.
2) Derive the susceptibility $\chi(\omega)$ and the power spectral density $I_\theta(\omega)$ for the fluctuations of $\theta$. 3) Derive the temporal correlation function $C_{\theta\theta}(t)$ of the angular fluctuations **in integral form** (no explicit expression asked).