M2 QLMN

Non Equilibrium Statistical Physics - TD 1

1 Polarizability Fluctuations for a Dielectric Particle

We consider a dielectric particle much smaller than the excitation wavelength. In presence of a static electric field $\mathcal{E}_0$, the particle has a dipole moment $\mathbf{p}_0$ given by its static susceptibility χ_0. When the electric field is switched off, the amplitude of the dipole moment decays exponentially with a time constant $\tau = 1/\gamma$.

Derive :
1. the relaxation function $\Psi(t)$ and the linear response $\chi(t)$
2. the complex susceptibility $\chi(\omega)$

2 Polarization of a Particle

We consider a system of volume V that can be polarized when an electric field is applied. The dipole moment of the system has a fixed mean value $\mathbf{p}$.

Deriving the probability of a state
Using Lagrange multipliers method, derive the probability of a state r of energy E_r and dipole moment p_r. Show that the probability can be cast in the form :

$$P_r = \frac{\exp[-\beta E_r + \alpha_x p_x + \alpha_y p_y + \alpha_z p_z]}{Z}.$$

Identifying the parameters α_i
1. Derive the entropy in the system.
2. Deduce from the entropy the expression of the generalized potential $A = -k_B T \ln Z$, Z being the partition function.
3. The interaction between the dipole moment and the electric field $\mathcal{E}$ contributes to the energy by the amount $-\mathbf{p} \cdot \mathcal{E}$. Derive the expression of α as a function of electric field and temperature.

Mean value and fluctuations
The objective here is to derive a relation between the fluctuations and the linear response using a partition function Z that will not be calculated.
4. Derive the mean value of the component p_i of the dipole moment as a function of Z.
5. Express the standard deviation of the dipole moment σ_{pi} as a function of Z.

Deriving the partition function
We assume that $\mathbf{p} \cdot \mathcal{E} << k_B T$.
6. Show that the partition function can be written as :

$$Z = Z_0[1 + \frac{\mathbf{p} \cdot \mathcal{E}}{k_B T} + \frac{(\mathbf{p} \cdot \mathcal{E})^2}{2k_B^2 T^2}], \tag{1}$$

where Z_0 is the partition function without external electric field.
7. The static susceptibility χ_0 is defined by $\mathbf{p} = \chi_0 \mathcal{E}$. Show that the standard deviation for the fluctuations of dipole moment is $\sigma_p^2 = k_B T \chi_0$.