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# TD 1 - Non-equilibrium Statistical Physics

# Linear response theory

## 1 Polarizability Fluctuations for a Dielectric Particle

We consider a dielectric particle much smaller than the excitation wavelength. In presence of a static electric field $\mathcal{E}_0$, the particle has a dipole moment $\mathbf{p}_0$ given by its static susceptibility $\chi_0$. When the electric field is switched off, the amplitude of the dipole moment decays exponentially with a time constant $\tau = 1/\gamma$. *Derive :*

1. the relaxation function $\Psi(t)$ and the linear response $\chi(t)$
2. the complex susceptibility $\chi(\omega)$

## 2 Polarization of a Particle

We consider a system of volume $V$ that can be polarized when an electric field is applied. The dipole moment of the system has a fixed mean value $\mathbf{p}$.

1) Using Lagrange multipliers method, derive the probability of a state $r$ of energy $E_r$ and dipole moment $p_r$. Show that the probability can be cast in the form :

$$P_r = \frac{\exp[-\beta E_r + \alpha_x p_x + \alpha_y p_y + \alpha_z p_z]}{Z}.$$

2) Derive the entropy in the system, and deduce from it the expression of the generalized potential $A = -k_B T \ln Z$, $Z$ being the partition function.

3) The interaction between the dipole moment and the electric field $\mathcal{E}$ contributes to the energy by the amount $-\mathbf{p} \cdot \mathcal{E}$. Derive the expression of $\alpha$ as a function of electric field and temperature.

*The objective now is to derive a relation between the fluctuations and the linear response using a partition function $Z$ that will not be calculated.*

4) Derive the mean value of the component $p_i$ of the dipole moment as a function of $Z$. Similarly, express the standard deviation of the dipole moment $\sigma_{pi}$ as a function of $Z$.

5) We assume that $\mathbf{p} \cdot \mathcal{E} << k_B T$. Show that the partition function can be written as :

$$Z = Z_0[1 + \frac{\mathbf{p} \cdot \mathcal{E}}{k_B T} + \frac{(\mathbf{p} \cdot \mathcal{E})^{\mathbf{2}}}{2k_B^2 T^2}], \quad (1)$$

where $Z_0$ is the partition function without external electric field.

6) The static susceptibility $\chi_0$ is defined by $\mathbf{p} = \chi_0 \mathcal{E}$. Show that the standard deviation for the fluctuations of dipole moment is $\sigma_p^2 = k_B T \chi_0$.